Stabilizing a Multimachine Power System via Output Feedback Excitation Control

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Abstract– *This paper presents a modified optimal controller for stabilizing a multimachine power system. The design method does not need the specification of weighting matrices. The eigenvalues of electromechanical modes would be shifted to a pre-specified vertical strip. For practical implementation, the proposed method design using an optimal reduced order model whose state variables are torque angles and speeds. The reduced order model retains their physical meaning and is used to design output feedback controller that takes into account the realities and constraints of the electrical power systems. Effectiveness of this controller is evaluated and example, a multimachine power system is given to illustrate the advantages and effectiveness of the proposed approach.*

Keywords– *dynamic stability, strip eigenvalue assignment, power system stabilizer, optimal reduced-order model, output feedback excitation control*

1. Introduction

The poor damping of electromechanical oscillation is symptomatic of intrinsic weaknesses in the power system. In some interconnections the situation is worsened by the growth of inter-utility wheeling, which is dictated by the economical constraints in modern power systems. These factors combine to bring the typical operating state closer than ever to the system stability limits and to make the damping of electromechanical oscillations a recurrent problem in the several power systems. Since the introduction of new control systems to the uncertain and multivariable environment of complex power systems is a slow process, which incurs a variety of risks, the full utilization of existing Power System Stabilizers (PSS) is essential for enhance the damping of low-frequency oscillations in the range 0.5 to 2 Hz, i.e. dynamic or steady-state stability of power systems [1-2]. Considerable efforts have been placed on the synthesis of power system stabilizers in multimachine power systems [3-9].

The design of PSS can be formulated as an optimal linear regulator control problem. However, the implementation of this technique requires the design estimators. This approach in increases the implementation cost and reduces the reliability of the control system. These are the reasons that a control scheme use only some desired state variables such as torque and speeds.

In recent years, the modal control design has been used in power systems to shift the dominant eigenvalues. Different methods have been proposed to assign eigenvalues by modifying the weighting matrix of the quadratic performance index. Optimal and sub-optimal control strategies on the basis of linear system theory using various system states and measurable output as input to the controller have also been attempted [10-11].

Although the closed-loop system constructed by using the optimal control theory has some advantages, they are still many problems to solve. One of the most serious is that it is rather difficult to specify the control performance described in terms of a quadratic performance index. The weighting matrices usually would be decided based on trial and error to give satisfactory performance. It is difficult to determine the weighting matrices of the performance index.

This paper presents a linear quadratic controller such that the optimal closed-loop system has eigenvalues lying within a vertical strip in the complex s-plane. Aiming at improving system stability which the design method does not need the specification of the weighting matrices. In this work, the desired positions of the eigenvalues are achieved without convergence problems. One basic difficulty of the state feedback control is that it is usually impractical since some of system states cannot be measured. An output feedback controller is preferred. The gains of the proposed controller are obtained from reduced order model and strip eigenvalue assignment.

The proposed method has been applied to two cases: one machine infinite bus system and a multimachine power system. The results of the study are presented to demonstrate the effectiveness of the proposed controller. A comparison between the performance of the proposed controller and the optimal reduced order model are also included.

The attractive futures of this paper are as follows:

(1) The optimal reduced order model is used to retain the physical meanings of the desired state variables and the deleted modes are optimized.
(2) By using strip eigenvalue assignment the desired positions of eigenvalues are achieved without convergence problem. The design method does not need the specification of the weighting matrices.
(3) The output state feedback is used to design the power system stabilizer and the type of the

controller is simple and easy to implement.

2. Optimal Reduced Order Model

The linear model of the electrical power system can be described by the following state space representation:

$$\dot{x} = Ax + Bu \tag{1}$$

where x and u are the nx1 state vector and mx1 input vector, respectively. A and B are constant matrices of appropriate dimensions.

Since the reduced order model derived in [13] is used in the following study, the process of evaluating the reduced order model is abbreviated as follows without proof.

The reduced order model is the derived using the following system whose first m variables are the desired variables z, which are speeds and torque angles in the proposed approach. The similarity transformation T is obtained in [13].

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u \tag{2}$$

$$z = [I_m, 0]\hat{x} \tag{3}$$

where

$$\hat{x} = Tx$$

$$\hat{A} = TAT^{-1}$$

$$\hat{B} = TB$$

I_m = m x m identity matrix

Assume that the eigenvalues of $\hat{A}$ are distinct this will actually be the case in the power system.

Let V = [V_1, V_2, . . ., V_n] where V_i is the right eigenvector of A associated with λ_i. Let $W = V^{-1}$,

Define $\phi = W\hat{x}$

Then

$$\dot{\phi} = \Lambda\phi + \Gamma u \tag{4}$$

$$z = D\phi \tag{5}$$

where

$\Lambda = W\hat{A}V =$ diagonal ($\lambda_1, \lambda_2, \ldots, \lambda_n$)

$$\Gamma = W\hat{B}$$

$$D = [I_m,\ 0]V$$

These equations can be arranged and written in partition form as:

$$\dot{\phi}_1 = \Lambda_1\phi_1 + \Gamma_1 u \tag{6}$$

$$\dot{\phi}_2 = \Lambda_2\phi_2 + \Gamma_2 u \tag{7}$$

$$z = D_1\phi_1 + D_2\phi_2 \tag{8}$$

where

Λ_1 contains modes to be retained

Λ_2 contains modes to be eliminated

Assume the reduced order system we are sought to determine will be of the form as follows:

$$\dot{z} = Fz + Gu \tag{9}$$

The evaluation algorithm of F and G proposed in [13] are abbreviated as follows:

$$F = D_1\Lambda_1 D_1^{-1} \tag{10}$$

Let V_m be the modal matrix associated with eqn.(10).
Define

$$\overline{F} = V_m^{-1}FV_m \tag{11}$$

$$\overline{G} = V_m^{-1}G \tag{12}$$

$$\overline{C} = V_m^{-1}D_2 \tag{13}$$

$$\overline{\Gamma}_1 = V_m^{-1}D_1\Gamma_1 \tag{14}$$

Then

$$\overline{S} = \overline{C}\Lambda_2 - \overline{F}\,\overline{C} \tag{15}$$

$$\Lambda = -(\overline{F} + \overline{F}^T)^{-1} \tag{16}$$

$$R = -\Lambda\overline{S} \tag{17}$$

Let $\alpha_i = \lambda_{m+i}$, i = 1, 2, . . . , n-m.

Then Λ_2 = diagonal ($\alpha_1, \alpha_2, \ldots, \alpha_{n-m}$)

The $(i,j)^{th}$ element of the mxp matrix is given by:

$$\Phi_{ij} = \frac{R_{ij}}{\lambda_i^* + \alpha_j} \tag{18}$$

where the subscript * denotes complex conjugate.

$$\Delta = \Lambda^{-1}\Phi \tag{19}$$

Let $\overline{K} = \overline{\Gamma}_1 + \overline{C}\Gamma_2$ (20)

Then $\overline{G} = \overline{K} + \Delta\Gamma_2$ (21)

And $G = V_m\overline{G}$ (22)

3. Strip Eigenvalue Assignment

Consider a linear time-invariant controllable system that is described in the state space by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{23}$$

$$y(t) = Cx(t) \tag{24}$$

where x(t), u(t), and y(t) are the nx1 state vector, mx1 input vector, and px1 output vector, respectively. A, B, and C are constant matrices of appropriate dimensions.

In the design of a conventionally optimal control system, the control vector is given by

$$u(t) = -Kx(t) \tag{25}$$

where K is the mxn state feedback control matrix designed to minimize the following quadratic performance index:

$$J = \tfrac{1}{2}\int_0^\infty (x^TQx + u^TRu)dt \tag{26}$$

In eqn.(26) the weighting matrices Q and R are nxn non-negative and mxm positive definite symmetric matrices, respectively. The feedback gain in eqn.(25) is K ($=R^{-1}B^TP$) with P being a symmetric positive definite matrix, which is solution of the following algebraic matrix Riccati equation,

$$A^TP + PA - PBR^{-1}B^TP + Q = 0_n \tag{27}$$

and the eigenvalues of A-BK, denoted by Λ(A-BK), will lie in the open left half plane of the complex s-plane.

In conventionally optimal system analysis, the gain in eqn.(25) is designed by roughly selecting weighting matrices according to physical reasoning. Because of complexity, the matrices Q and R are commonly chosen as diagonal matrices.

The eigenvalues of the closed-loop system are denoted by Λ(A-BK) = [λ_1, λ_2, ..., λ_m, λ_{m+1}, ..., λ_n]. In order to improve the system performance, the eigenvalues λ_1 through λ_m will be selected and shifted to a desired region. To achieve this results the weighting matrix R in the eqn.(27) is set to be an identity matrix for equal weighting of the m control inputs, and the weighting matrix Q must be given, but in large power system, it is not easy to determine those weighting matrices. The weighting matrices usually are determined by trial and error to obtain satisfactory performances. To overcome this difficulty, a novel approach for designing the optimal eigenvalues assignment will be proposed in the following discussion. The design method in this paper shifts the closed-loop eigenvalues to a pre-specified vertical strip without the need of weighting matrices.

Let (A, B) be the pair of the open-loop system matrices in eqn.(23) and h ≥ 0 represent the prescribed degree of relative stability. Then the closed-loop matrix $A_c = A - BR^{-1}B^T\tilde{P}$ has all its eigenvalues lying on the left side of the -h vertical line as shown in Fig. 1(a), where the matrix $\tilde{P}$ is the solution of the following Riccati equation [12];

$$(A + hI_n)^T\tilde{P} + \tilde{P}(A + hI_n) - \tilde{P}BR^{-1}B^T\tilde{P} + Q = 0_n \qquad (28)$$

Note that in eqn.(28) with Q = 0_n, the unstable eigenvalues of (A + hI_n) are shifted to their mirror image positions with respect to the -h vertical lie, which are the eigenvalues of the closed-loop system matrix A_c.

Assume that h_1 and h_2 are two positive real values to determine an open vertical strip of [-h_2, -h_1] on the negative real axis as shown in Fig. (1b) and give an nxn matrix $\tilde{A} = A + h_1I$.

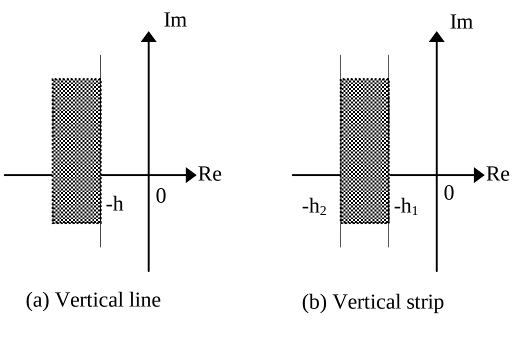

Fig. 1 Complex s-plane

The control law changed to be

$$u(t) = -\rho\tilde{K}x(t) \qquad (29)$$

with the feedback gain $\tilde{K} = R^{-1}B^T\tilde{P}$. The matrix $\tilde{P}$ is the solution of the following modified Riccati equation:

$$\tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} - \tilde{P}BR^{-1}B^T\tilde{P} = 0_n \qquad (30)$$

The gain $\rho$is selected by

$$\rho = \frac{1}{2} + \frac{h_2 - h_1}{2.\mathrm{tr}(\tilde{A}^+)} = \frac{1}{2} + \frac{h_2 - h_1}{\mathrm{tr}(B\tilde{K})} \qquad (31)$$

where $\mathrm{tr}(\tilde{A}^+) = \sum_{i=1}^{n^+}\lambda_i^+ = \frac{1}{2}\mathrm{tr}(B\tilde{K})$ and $\lambda_i^+ (i = 1,2,\cdots,n^+)$ are the eigenvalues of $\tilde{A}$ in the right half-plane of the complex s-plane. The optimal closed-loop system becomes

$$\dot{x}(t) = (A - \rho B\tilde{K}) \qquad (32)$$

Eqn.(32) consists of a set of eigenvalues which lie inside the vertical strip of the [-h_2, -h_1] as shown in Fig. (1b). In eqn.(30) for equal weighting of the m control inputs, we can let R be unity matrix. These solving the Riccati eqn.(30) does not need a Q matrix, so it is easy to design an optimal controller for power system oscillation damping.

4. Output Feedback Excitation Control Design

To avoid the drawback demonstrated in the above section; we should use the optimal reduced order model derived in [13] to retain the physical meanings of the output states which also the entries in the strip eigenvalue assignment we are interested in. By using the reduced order model, the system in eqn.(1) can be reduced to the following form:

$$\dot{x}_r = A_rx_r + B_ru^* \qquad (33)$$

where

- $x_r \in R^{mx1}$: state vector to be retained consisting of torque angles and speeds in electric power system.
- A_r , B_r : constant matrices of reduced order model with appropriate dimensions.

The control law can be written to the following form:

$$u^* = -\rho K_rx_r \qquad (34)$$

with the feedback gain $K_r = R^{-1}B_r^T\tilde{P}$. The matrix $\tilde{P}$ is solution of the following modified Riccati equation

$$\tilde{A}_r^T\tilde{P} + \tilde{P}\tilde{A}_r - \tilde{P}B_rR^{-1}B_r^T\tilde{P} = 0 \qquad (35)$$

where $\tilde{A}_r = A_r + h_1I$. The gain $\rho$is selected by using the expressions given in section 3.

Fig. 2 illustrates how the theory of the above regulator able to implemented by a PI controller.

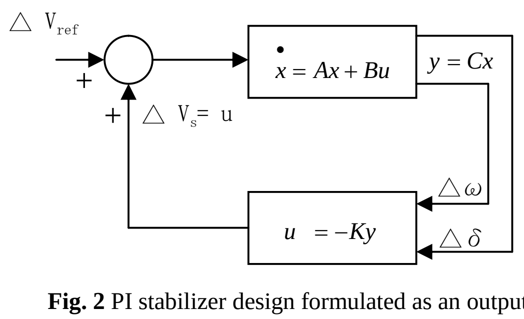

Fig. 2 PI stabilizer design formulated as an output feedback regulator problem

5. Simulation Results

To assess the proposed method in the case of multimachine system, the system shown in the Fig. 3, taken from [23], is studied.

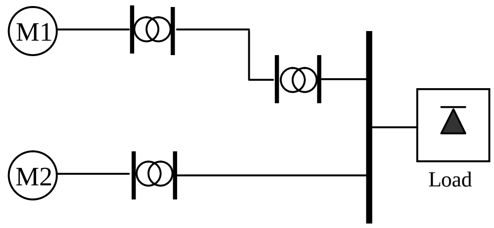

Fig. 3 Multimachine system

The model given in [23] is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{36}$$

where

$$x^T = [\Delta\omega_1\ \Delta\delta_1\ \Delta e'_{q1}\ \Delta e_{FD1}\ \Delta\omega_2\ \Delta\delta_2\ \Delta e'_{q2}\ \Delta e_{FD2}]$$

$$A = \begin{bmatrix} -0.244 & -0.0747 & -0.1431 & 0 & 0 & 0.0747 & 0.0041 & 0 \\ 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.046 & -0.455 & 0.244 & 0 & 0.046 & 0.13 & 0 \\ 0 & -398.56 & -19498.8 & -50 & 0 & 398.58 & -3967 & 0 \\ 0 & 0.178 & -0.0433 & 0 & -0.2473 & -0.178 & -0.146 & 0 \\ 0 & 0 & 0 & 0 & 376.99 & 0 & 0 & 0 \\ 0 & 0.056 & 0.1234 & 0 & 0 & -0.0565 & -0.3061 & 0.149 \\ 0 & -677.39 & -10234.22 & 0 & 0 & 677.78 & -13364.16 & -50 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 25000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25000 \end{bmatrix}^T$$

The eigenvalues of the system as shown in **Fig. 3** without control are listed in the first column of Table 1. The first pair of eigenvalues is electromechanical mode. It can be observed from Table 1 that the minimal damping ratio of electromechanical mode is 0.0092, so that it is not satisfactory. To improve the system dynamic performance, this mode should be shifted toward certain desirable location. In the proposed method, if we choose h_1 = 3.0 and h_2 = 3.5, the electromechanical and other modes with absolute real parts less than h_1 = 3.0, will be shifted to the vertical strip of [$-h_2$, $-h_1$] = [-3.5, -3.0]. The other modes will not be changed (see, subsection 5.1). The minimal damping ratio of that mode is improved to be 0.267 in this proposed method, that its inside the acceptable range [1-2]. Two output feedback schemes are compared: (1) optimal reduced order model, and (2) proposed method. It is shown from Table 1, that the relative stability of the proposed method is much better than optimal reduced order model [23]. The feedback gains are given in Table 2. The transient responses of the angular frequencies to a 5 % change in the mechanical torque of machine 1 and machine 2 are shown in Fig. 4 and Fig. 5, respectively

Table 1 System eigenvalues

Open-Loop	Reduced Order Model	Proposed Method
-0.0904±j9.8430 -0.0006 -0.2443 -25.1741±j67.8187 -25.2329±j30.3073	-0.6120±j10.2843 -1.9248±j1.9185 -23.3329±j67.2307 -24.9273±j29.8745	-3.2634±j11.7720 -3.1648±j2.8601 -21.8258±j66.6676 -22.3723±j27.3902

Vertical strip in h_2 = 3.5 ; h_1 = 3.0

Table 2 Feedback gains

	Reduced Order Model		Proposed Method	
	u_1	u_2	u_1	u_2
$\Delta\omega_1$	196.5413	32.4768	36.6213	4.0898
$\Delta\delta_1$	1.2387	0.1697	0.3751	0.0918
$\Delta\omega_2$	59.4160	0.3957	6.3898	16.3856
$\Delta\delta_2$	0.1028	.3081	0.0570	0.0912

ρ = 0.5216

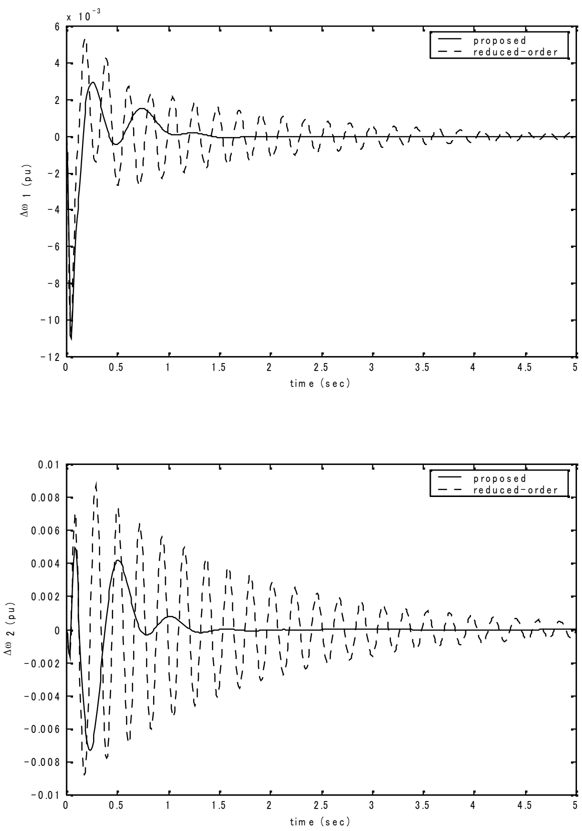

Fig. 4 Transient responses of the angular frequencies to a 5 % change in the mechanical torque of machine 1

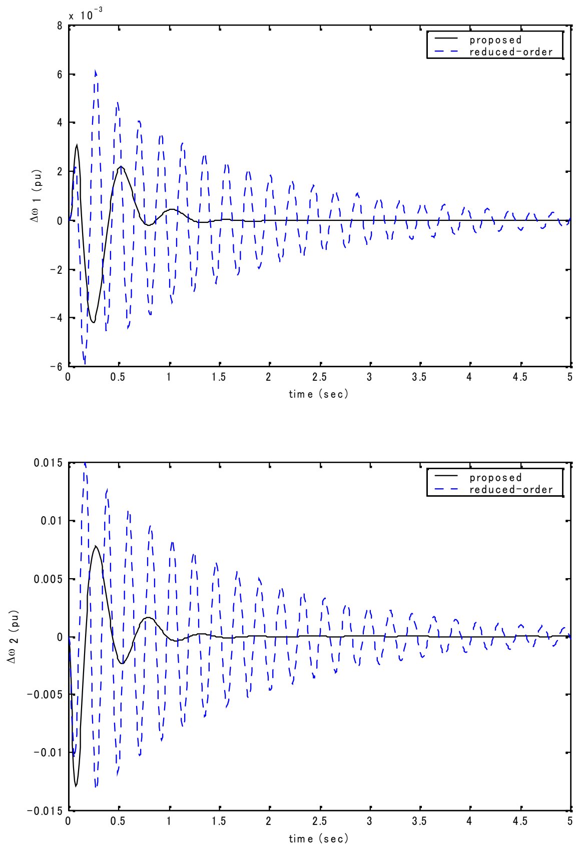

Fig. 5 Transient responses of the angular frequencies to a 5 % change in the mechanical torque of machine 2

6. Conclusions

Stabilizing a multimachine power system is achieved using an output feedback excitation control. The reduced order model retains the modes that mostly affect some desired variables which are usually the variable or measurable variables. In this analysis these variables are torque angles and angular frequencies (speeds). The electromechanical mode can be shifted to a pre-specified vertical strip without effect the other modes. Starting with the optimal reduced order model approach, the algorithm for the designing output feedback excitation controller is constructed. The design method is very simple and avoids the difficulty of choosing weighting matrices. Simulation results indicate that the proposed controller provides an effective means for improving the damping characteristics of the power system.

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